Daxiu Wei*, Xiaodong Yang, Jun Luo, Xianping Sun, Xizhi Zeng, Maili Liu, and Shangwu Ding
Laboratory of Magnetic Resonance and Atomic and Molecular Physics, Wuhan Institute of Physics and Mathematics, The Chinese Academy of Sciences, Wuhan 430071, People's Republic of China

The controlled-NOT gate and controlled square-root NOT gate play an important role in quantum algorithm. This article reports the experimental results of these two universal quantum logic gates (controlled square-root NOT gate and controlled-NOT gate) on a 7-qubit NMR quantum computer. Further, we propose a simple experimental method to measure and correct the error in the controlled phase-shift gate, which is helpful to construct a more perfect phase-shift gate experimentally and can also be used in more qubits discrete Fourier transformation.

PACS number(s): 03.67.Lx. 76.60.-k. 75.10.Jm

I. INTRODUCTION

By using the characteristics of quantum mechanics, quantum computers (QC) are faster than classical computers when performing certain computations such as the factorization of a large number [1], searching of database [2] especially when simulating quantum systems [3]. Among many schemes of realizing quantum computers (for example trapped ions [4], cavity QED [5] , quantum dots [6], NMR [7,8]), the scheme based on liquid NMR techniques has made remarkable progress [9]. In experiments, the four-qubit entanglement has been implemented by using trapped ions [10], and seven-qubit cat-state [11], five-qubit D-J algorithm [12], order finding algorithm [13] by using liquid NMR techniques. People are most interested in extension quantum computation to multi-qubit spin systems [14,15].

However, it is a technical challenge to extend to more qubits experimentally due to the low signal-to-noise (S/N) ratio of NMR and further, the S/N ratio will decrease exponentially with the increasing of qubit [16]. Hence, the quantum computing based on liquid NMR will reach no more than 10 qubits [16]. With the increasing of the number of qubit, it is not easy to selectively control the coherent evolution between two specific spins as the coupling network becomes more complex. Though, efficient methods have been developed to refocus the interaction of single-spin and two spins [17]. In a multi-qubit NMR QC, it is necessary to suppress the interactions coming from the other qubits when realizing the quantum logic gate between two given spins [17]. Moreover, in the process of achieving complicated quantum computation, the precision of every gate plays a very important role in realizing quantum computing [18]. It is a big technical challenge if we use homonuclear system to realize multi-qubit computing, due to the use of selective pulse sequences. So, it is necessary to establish a set of measurement to examine and correct the quantum logic gate.

The controlled-NOT (CNOT) gate and controlled square-root NOT gate [9] play an central role in many quantum computations (especially quantum Fourier Transformation [19], the Grover's searching algorithm [2], the Toffoli gate [20]). In this article, we report the experimental realization of controlled-NOT gate and controlled square-root NOT gate between two given qubits on seven-qubit NMR quantum computers, and we also propose a method to verify and correct the phase-shift error.

II. PERFECT CONTROLLED-NOT GATE AND CONTROLLED SQUARE-ROOT NOT GATE (UNIVERSAL PHASE-SHIFT GATE)

CNOT gate is a kind of universal controlled phase-shift gate, which is defined by [9]:

$$| 1\rangle \, | 1\rangle \xrightarrow{\varphi} e^{i\varphi} \, | 1\rangle \, | 1\rangle. \qquad (1)$$

The quantum circuit to realize a controlled square-root NOT gate is shown in figure 1. h and h $^{-1}$are pseudo-Hadamard gates, which can be implemented by 90 degree pulses along y axis. The phase-shift gate Z_{Φ} of any angle can be realized by selective composite z-pulses:

*Electronic mail: daxiuwei@wipm.whcnc.ac.cn

$$\frac{1}{4nJ_{ij}} - 180^\circ_x - \frac{1}{4nJ_{ij}} - 90^\circ_x - (\frac{\varphi}{2})_{-y} - 90^\circ_{-x}. \tag{2}$$

Pulses here affects two given spins i and j simultaneously. When n=1, the logic gate according to figure.1 is a controlled-NOT gate. When n=2, it is a controlled square-root NOT gate. When n is an arbitrary integer, it is a controlled n-th square-root NOT gate.

We select a seven-qubit spin system (^{13}C-labeled trans-crotonic acid ($C^1H^3_3\ C^2H^1{=}C^3H^2C^4O_2H$) [11]), and begin the experiment from a thermal equilibrium state in a product operator representation, we then measure the phase-shift Φ and correct it as follow, which is simple and feasible in the experiment:

$$\begin{aligned}
&\mu_C(I_z^{C^1} + I_z^{C^2} + I_z^{C^3} + I_z^{C^4}) + \mu_H(I_z^{H^1} + I_z^{H^2} + I_z^{H^3}) \\
&\xrightarrow{(\pi/2)^{C^2}_{-y}} \mu_C(I_z^{C^1} + I_x^{C^2} + I_z^{C^3} + I_z^{C^4}) + \mu_H(I_z^{H^1} + I_z^{H^2} + I_z^{H^3}) \\
&\xrightarrow{1/2nJ_{C^1C^2}} \mu_C(I_z^{C^1} + I_x^{C^2}\cos\frac{\pi}{2n} - 2I_y^{C^2}I_z^{C^1}\sin\frac{\pi}{2n} + I_z^{C^3} + I_z^{C^4}) + \mu_H(I_z^{H^1} + I_z^{H^2} + I_z^{H^3}) \\
&\xrightarrow{(\pi/2)^{C^2}_X} \mu_C(I_z^{C^1} + I_x^{C^2}\cos\frac{\pi}{2n} + 2I_z^{C^2}I_z^{C^1}\sin\frac{\pi}{2n} + I_z^{C^3} + I_z^{C^4}) + \mu_H(I_z^{H^1} + I_z^{H^2} + I_z^{H^3}) \\
&\xrightarrow{PFG} \mu_C(I_z^{C^1} + 2I_z^{C^2}I_z^{C^1}\sin\frac{\pi}{2n} + I_z^{C^3} + I_z^{C^4}) + \mu_H(I_z^{H^1} + I_z^{H^2} + I_z^{H^3}) \\
&\xrightarrow{(\pi/2)^{C^2}_X} \mu_C(I_z^{C^1} + 2I_y^{C^2}I_z^{C^1}\sin\frac{\pi}{2n} + I_z^{C^3} + I_z^{C^4}) + \mu_H(I_z^{H^1} + I_z^{H^2} + I_z^{H^3}) \\
&\xrightarrow{(\Phi)^{C^2}_z} \mu_C(I_z^{C^1} + 2I_y^{C^2}I_z^{C^1}\sin\frac{\pi}{2n}\cos\Phi + 2I_x^{C^2}I_z^{C^1}\sin\frac{\pi}{2n}\sin\Phi + I_z^{C^3} + I_z^{C^4}) + \mu_H(I_z^{H^1} + I_z^{H^2} + I_z^{H^3}),
\end{aligned} \tag{3}$$

C^2 here denotes selective pulses on Carbon 2. $1/2nJ_{C^1C^2}$ is J-coupling evolution. μ_C and μ_H are the nuclear magnetic moments. PFG denotes the pulsed field of gradient. $(\Phi)_z^{C^2}$ expresses selective composite z-pulses on C^2 , and the refocusing and decoupling pulses are not given here.

We have used the following two steps to measure the phase shift Φ :

(1). Add a selective read-pulse $(\frac{\pi}{2})_x^{C^2}$ behind the above pulses, the product operator is :

$$\mu_C(I_z^{C^1} - 2I_z^{C^2}I_z^{C^1}\sin\frac{\pi}{2n}\cos\Phi + 2I_x^{C^2}I_z^{C^1}\sin\frac{\pi}{2n}\sin\Phi + I_z^{C^3} + I_z^{C^4}) + \mu_H(I_z^{H^1} + I_z^{H^2} + I_z^{H^3}), \tag{4}$$

here the signal is $2I_x^{C^2}I_z^{C^1}\sin\frac{\pi}{2n}\sin\Phi$.

(2). Add a selective read-pulse $(\frac{\pi}{2})_y^{C^2}$behind the above pulses, the product operator is :

$$\mu_C(I_z^{C^1} + 2I_y^{C^2}I_z^{C^1}\sin\frac{\pi}{2n}\cos\Phi + 2I_z^{C^2}I_z^{C^1}\sin\frac{\pi}{2n}\sin\Phi + I_z^{C^3} + I_z^{C^4}) + \mu_H(I_z^{H^1} + I_z^{H^2} + I_z^{H^3}), \tag{5}$$

here the signal is $2I_y^{C^2}I_z^{C^1}\sin\frac{\pi}{2n}\cos\Phi$.

By repeatedly adjusting the phases of the receiver, we obtain the integrated intensities of the two experimental signals. The phase-shift Φ can be calculated from the ratio $\tan\Phi$. Then the Φ of the composite pulses and the evolution time are adjusted to correct the error so as to obtain a more accurate controlled square-root NOT gate.

III. EXPERIMENT AND RESULT

We carried out the experiment with a Bruker-ARX500 spectrometer. The sample was 20mg crotonic acid (U-$^{13}C_4$, Cambridge Isotope Laboratories Inc. Cat. No. CLM-6118) dissolved in deuterated acetone, which was degassed and flame-sealed in a standard 5mm NMR test tube. The experimental temperature was maintained at 25°C. The molecular structure of crotonic acid can be found in [11]. We have used four carbons and three protons as the seven qubits. The shape of the soft pulse is Gaussian. Typical experiment results are shown in figures 2 and 3. Both are C^2 NMR spectra (not decoupled). The theoretical and experimental results of Φ in the controlled square-root NOT gate, and the relative errors $|\frac{\Phi_{\rm exp}-\Phi_{\rm the}}{\Phi_{\rm the}}|$ between the theoretical phase-shift $\Phi_{\rm the}$ and experimental $\Phi_{\rm exp}$ are shown in table 1. The largest error is 3.2%. Using the method in the last section, where we have adjusted the parameters of the Gaussian pulses and the evolution time, we achieved to control the precision of the controlled-NOT gate and realized a more perfect conditional-phase-shift gate. The experimental errors mainly come from: (1) the inhomogeneity of the static and radio-frequency magnetic field, (2) the imperfection of the selective pulse, (3) the inaccuracy of the 90 degree pulse, and (4) the decaying of signal during the acquiring time.

IV. CONCLUSION

We realized the controlled-NOT and controlled square-root NOT gates between two given qubits on a seven-qubit NMR quantum computer. The experimental result shows that the control of the other qubit's evolution is successful. We also give a method to improve the accuracy of the phase-shift gate, which can be applied to many quantum algorithms, such as multi-qubit Fourier Transformation. Besides, it is also possible to implement this, the method of measuring and correcting the error of the controlled square-root NOT gate as discussed before, into the design of other logic gates in a multi-qubit quantum computer.

V. ACKNOWLEDGMENTS

This work has been supported by the National Natural Science Foundation of China (Grant No.19874073), the National Knowledge Innovation Program of the Chinese Academy of Sciences (Grant No. KJCX2-W1). We thank Dr. Xi-an Mao, and Ms. Hanzhen Yuan for valuable discussions and technical assistance.

[1] P. Shor, in Proceedings of the 35th Annual Symposium on Foundations of Computer Science (IEEE Computer Society, Los Alamitos, CA, 1994), pp.124-134.
[2] L. Grover, in Proceedings of the 28th Annual ACM Symposium on the Theory of Computing (Philadelphia, Pennsylvania, 22-24 May 1996), pp. 212-219.
[3] S. Lloyd, Science **273**, 140 (1996).
[4] J. Cirac and P. Zoller, Phys. Rev. Lett. **74**, 4091 (1995).
[5] Q.A. Turchette *et al.*, Phys. Rev. Lett. **75**, 4710 (1995).
[6] S. Bandyopadhyay and V. Roychowdhury, Jpn. J. Appl. Phys. **35**, 3350 (1996).
[7] N. Gershenfeld and I. L. Chuang, Science **275**, 350 (1997).
[8] D. G. Cory, A. F. Fahmy, and T. F. Havel, Proc, Natl. Acad. Sci. U.S.A. **94**,1634 (1997).
[9] J. A. Jones, Prog. NMR. Spectrosc. **38**, 325 (2001).
[10] C.A. Sackett *et al.*, Nature **404**, 256 (2000).
[11] E. Knill, R. Laflamme, R. Martinez, and C.-H. Tseng, Nature **404**, 368 (2000).
[12] R. Marx, A. F. Fahmy, J.M. Myers, W. Bermel, S. J. Glaser, Phys. Rev. A **62**, 012310 (2000).
[13] L.M.K.Vandersypen, M. Steffen, G. Breyta, C.S.Yannoni, R. Cleve, and I. L.Chuang, Phys. Rev. Lett. **85**, 5452 (2000).
[14] G. P. Berman, G. D. Doolen and V. I. Tsifrinovich, Superlatt. Microstruct. **27**, 89 (2000).
[15] J. I. Cirac and P. Zoller, Nature **404**, 579 (2000).
[16] W. S. Warren, Science **277**, 1688 (1997).
[17] J. A. Jones, and E. Knill, J. Magn. Reson. **141**, 322 (1999).
[18] E. Knill, R. Laflamme, W. H. Zurek, Science **279**, 342 (1998).
[19] D. Coppersmith, IBM Research Report No. RC19642, (1994).
[20] D. P. DiVincenzo, Proc. R. Soc. Lond. A **454**, 261 (1998).

Table 1. The comparison between Φ_{the} and Φ_{exp}.

Φ_{the}	$2I_x^{C^2}I_z^{C^1}\sin\frac{\pi}{2n}\sin\Phi$	$2I_y^{C^2}I_z^{C^1}\sin\frac{\pi}{2n}\cos\Phi$	$\tan\Phi$	Φ_{exp}	$\lvert\frac{\Phi_{\text{exp}}-\Phi_{\text{the}}}{\Phi_{\text{the}}}\rvert$ (%)
90°	1.00	0.02	50	88.9°	1.3
180°	0.09	-1.00	-0.09	175.5°	2.7
270°	-1.00	0.05	19.5	267.1°	3.3
360°	0.07	1.00	0.07	356.3°	1.0

Figure captions

Fig. 1: A quantum circuit for implementing a controlled square-root NOT gate, C^1 the controlled qubit and C^2 the target qubit.

Fig. 2: The spectra of C^2. Both spectra were acquired with 8 scans. Results of realizing a controlled-NOT gate (a) and the thermal equilibrium state (b). The experimental results are in good agreement with the theoretical.

Fig. 3: The spectra of realizing a controlled square-root NOT gate. Both spectra were acquired with 8 scans. Signals of $2I_x^{C^2}I_z^{C^1}\sin\frac{\pi}{2n}\sin 270°$ (a) and $2I_y^{C^2}I_z^{C^1}\sin\frac{\pi}{2n}\cos 270°$ (b).